

# Resonant Frequencies, $Q$ -Factor, and Susceptance Slope Parameter of Waveguide Circulators Using Weakly Magnetized Open Resonators

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**Abstract**—A useful quantity in the description of junction circulators is the difference between the split frequencies of the magnetized ferrite resonator. A knowledge of this quantity allows the loaded  $Q$ -factor of a junction using a weakly magnetized resonator to be determined. This paper derives an exact description of the former quantity in the case of the open quarter-wave long (partial-height) disk resonator used in the construction of commercial turnstile waveguide circulators. This is done by employing duality between a ferrite-filled circular waveguide having ideal electric wall boundary conditions and one having ideal magnetic wall boundaries. The effect of an image wall on the open flat face of the open resonator is considered separately. The paper includes some remarks about the susceptance slope parameters of disk and triangular open resonators.

## I. INTRODUCTION

A USEFUL MODEL of a junction circulator is in terms of a magnetized ferrite or garnet resonator symmetrically coupled by three transmission lines. An important quantity in the synthesis of this class of device is its loaded  $Q$ -factor. For a weakly magnetized junction it is completely determined by the frequencies of the magnetized and demagnetized resonator. The mode spectrum of magnetized resonators for use in the realization of junction circulators is therefore of considerable interest.

An important class of commercial waveguide circulators is that using quarter-wave coupled quarter-wave long open triangular or circular resonators open-circuited at one end and short-circuited at the other, or half-wave long open resonators open-circuited at both ends [1]–[6]. Fig. 1 depicts schematic diagrams of three typical arrangements using disk resonators. Introduction of an image plane in the configurations in Fig. 1(a) and (b) indicates that they are dual in that a single set of variables may be used to describe both geometries. The device in Fig. 1(c) is also equivalent to the former ones, except that its susceptance slope parameter is twice that of the other two [6]. A quarter-wave long magnetized ferrite resonator short-circuited at one end, and open-circuited or loaded by an image wall at the other, is therefore a suitable prototype for the construction of this class of device. Although the mode

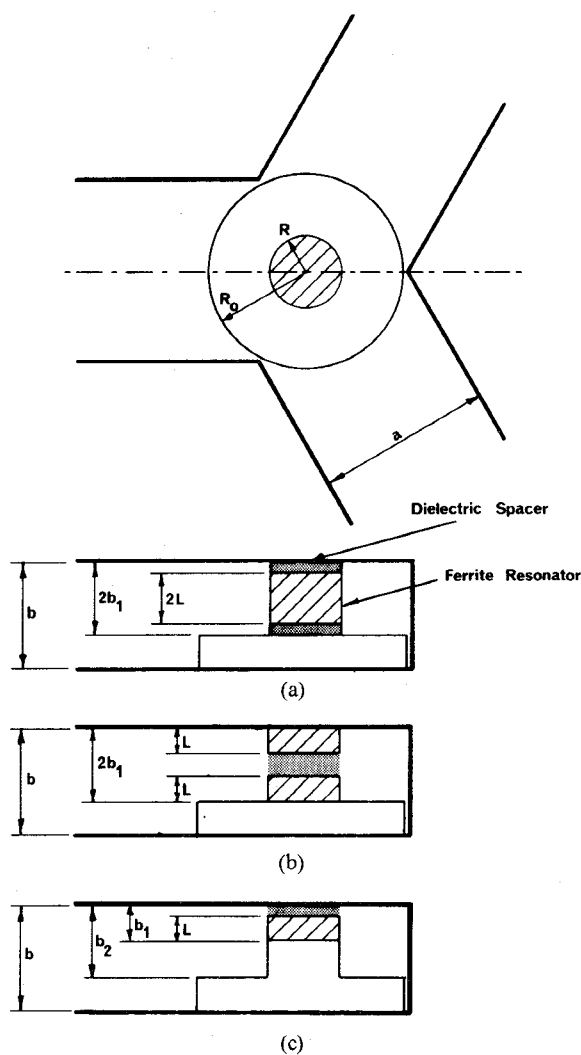


Fig. 1. Schematic diagrams of waveguide circulators using partial-height disk resonators.

spectrum of this type of resonator has been understood for some time [4], [5], only the resonant frequencies of the demagnetized disk and triangular resonators have been determined in closed form [6]–[8], [19]. This paper gives an exact derivation of magnetized disk resonators with the open flat face idealized by a magnetic or an image wall.

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This is done by employing duality between a ferrite-filled circular waveguide with electric wall boundaries, and one with ideal magnetic walls. A complete modal description has also been recently utilized to evaluate the overall reflection coefficient of this class of circulator but the split frequencies of the magnetized resonator have not been implicitly determined [16].<sup>1</sup>

The experimental results in this paper indicate that junction circulators using weakly magnetized open-disk and triangular ferrite resonators exhibit similar relationships between the magnetization of the resonator and its loaded  $Q$ -factor. The choice of resonator shape is therefore primarily determined by its susceptance slope parameter. However, the ripple level in the circulator specification has a significant influence on the impedance level of the gyrator circuit so that the resonator configuration is not as critical as previously supposed [14], [15]. Since a knowledge of the loaded  $Q$ -factor and the susceptance slope parameter is sufficient for the synthesis of this class of circulator, some remarks about the latter quantity for disk and side and apex coupled triangular resonators are included for completeness.

## II. SPLIT FREQUENCIES OF QUARTER-WAVE LONG OPEN-DISK RESONATORS

Circulators using weakly magnetized resonators for which the in-phase eigennetwork may be idealized by a short-circuit boundary condition, exhibit 1-port equivalent gyrator conductances at their operating frequency which may be described by

$$g = \sqrt{3} b' \left( \frac{\omega_+ - \omega_-}{\omega_0} \right). \quad (1)$$

$g$  is the normalized gyrator conductance,  $b'$  is the normalized susceptance slope parameter of the complex gyrator circuit, and  $\omega_0$  and  $\omega_{\pm}$  are the operating frequency of the circulator and the split frequencies of the magnetized resonator. Any two of the above variables are sufficient to define the gyrator equation.

In the case of waveguide circulators using open partial-height disk resonators, analytical descriptions of these quantities are still somewhat incomplete. However, some experimental data [17] and one approximation [8] are available on the split frequencies, and some experimental and semiempirical data is available on the susceptance slope parameter [9]. A knowledge of the split frequencies of such a resonator also leads to the description of its loaded  $Q$ -factor and to nearly exact synthesis of this class of circulator.

The relationship between the off-diagonal component of the permeability tensor and the ratio of the difference between the split frequencies of the magnetized and that of the demagnetized open-ferrite resonator will now be de-

termined. This is done assuming that the resonator consists of a quarter-wave long demagnetized or magnetized ferrite waveguide with ideal-magnetic walls open-circuited at one end and short-circuited at the other

$$\cot(\beta_0 L_0) = 0 \quad (2)$$

$$\cot(\beta_{\pm} L_0) = 0. \quad (3)$$

The first of these two equations determines the length of the open resonator from a knowledge of  $k_0 R$  and frequency

$$\beta_0 L_0 = \frac{\pi}{2} \quad (4)$$

where

$$\beta_0^2 = \left( k_0 \sqrt{\epsilon_f \mu_{\text{eff}}} \right)^2 - \left( \frac{1.84}{R} \right)^2 \quad (5)$$

and

$$k_0 = \frac{\omega}{c}. \quad (6)$$

$L_0$  is the length of the open resonator (m),  $R$  is the radius of the resonator (m),  $k_0$  is the free-space wavenumber (rad/m),  $\epsilon_f$  is the relative dielectric constant of the garnet or ferrite resonator,  $\mu_{\text{eff}}$  is the relative permeability of the magnetized garnet or ferrite resonator,  $\omega$  is the radian frequency (rad/s), and  $c$  is the free-space velocity ( $3 \times 10^8$  m/s).

The second boundary condition may be solved for the relationship between the split frequencies of the resonator in the neighborhood of the demagnetized one and the magnetic variables by forming the characteristic equation for  $\beta_{\pm}$  and using the boundary condition in (3) with  $L_0$  fixed by (4)

$$\beta_{\pm} L_0 = \frac{\pi}{2}. \quad (7)$$

The split phase constants  $\beta_{\pm}$  may be exactly evaluated using duality between a magnetized ferrite filled circular waveguide with ideal electric wall boundary conditions and one having ideal magnetic walls. The former problem is a classic result whose solution is given as [10]–[12]

$$\left( \frac{k_0^2 \epsilon_f - \beta^2}{k_1} - k_1 \right) \frac{J'_n(k_1 R)}{J_n(k_1 R)} - \left( \frac{k_0^2 \epsilon_f - \beta^2}{k_2} - k_2 \right) \frac{J'_n(k_2 R)}{J_n(k_2 R)} - \frac{\kappa}{\mu} \frac{n}{R} \beta^2 \left( \frac{1}{k_1^2} - \frac{1}{k_2^2} \right) = 0 \quad (8)$$

$$k_1^2 = k_0^2 \epsilon_f \left[ 1 - \frac{1}{2} \left( \frac{\kappa}{\mu} \right)^2 \right] - \beta^2 + \frac{\kappa}{\mu} \left[ k_0^2 \epsilon_f \left( \beta^2 + \frac{1}{4} k_0^2 \epsilon_f \left( \frac{\kappa}{\mu} \right)^2 \right) \right]^{1/2} \quad (9)$$

$$k_2^2 = k_0^2 \epsilon_f \left[ 1 - \frac{1}{2} \left( \frac{\kappa}{\mu} \right)^2 \right] - \beta^2 - \frac{\kappa}{\mu} \left[ k_0^2 \epsilon_f \left( \beta^2 + \frac{1}{4} k_0^2 \epsilon_f \left( \frac{\kappa}{\mu} \right)^2 \right) \right]^{1/2}. \quad (10)$$

The magnetic variables, assuming a saturated material,

<sup>1</sup>A recent paper, not available at the time of writing, giving theoretical data on the split frequencies of partial-height resonators in radial cavities is given in [20].

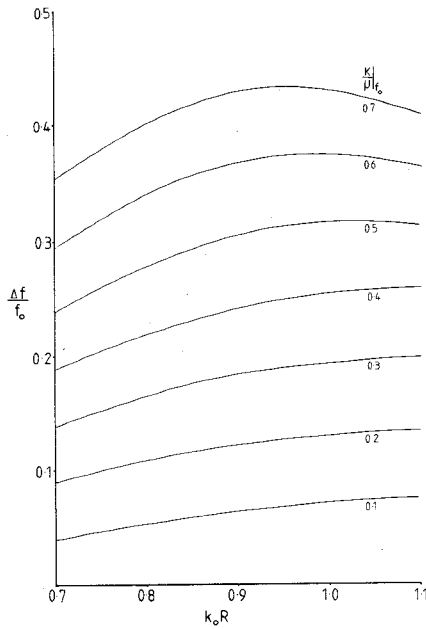


Fig. 2. Split frequencies of magnetized open-ferrite disk resonator.

are defined in the usual way by

$$\mu = 1 \quad (11)$$

$$\kappa = \frac{\gamma M_0}{\mu_0 \omega} \quad (12)$$

$$\mu_{\text{eff}} = 1 - \kappa^2. \quad (13)$$

$M_0$  is the saturation magnetization (Tesla),  $\mu_0$  is the free-space permeability ( $4\pi \times 10^{-7}$  H/m),  $\gamma$  is the gyromagnetic ratio ( $2.21 \times 10^5$  (rad/s)/(A/m)), and  $\mu$  and  $\kappa$  are the relative diagonal and off-diagonal components of the tensor permeability.

Equations (9) and (10) may occasionally have imaginary roots which require modified Bessel functions in equation (8) [11].

Equation (8) may be solved for  $\beta_{\pm}$  or  $k_{\pm}$  with  $n = \pm 1$  subject to the boundary condition in (7). The calculation indicates that the splitting between the degenerate modes is a function of  $k_0 R$  (Fig. 2). The result for a typical value of  $k_0 R$  equal to 0.8 is

$$\frac{\omega_+ - \omega_-}{\omega_0} \approx 0.0656 \left( \frac{\kappa}{\mu} \right)^3 - 0.061 \left( \frac{\kappa}{\mu} \right)^2 + 0.621 \left( \frac{\kappa}{\mu} \right), \quad (14)$$

$$0 \leq \left( \frac{\kappa}{\mu} \right) \leq 0.5.$$

Fig. 3 gives one experimental plot of the split frequencies for a disk resonator using a garnet material with a dielectric constant of 15.3, a magnetization of 0.1760 T, for which  $k_0 R = 0.82$  at 9 GHz. The lack of symmetry in the splitting at magnetic saturation is partly due to the form of  $\kappa$  in (12). The resonator is in direct contact with one waveguide wall to minimize the effect of the image wall on the result. The agreement between theory and experiment is in this instance within 3 percent.

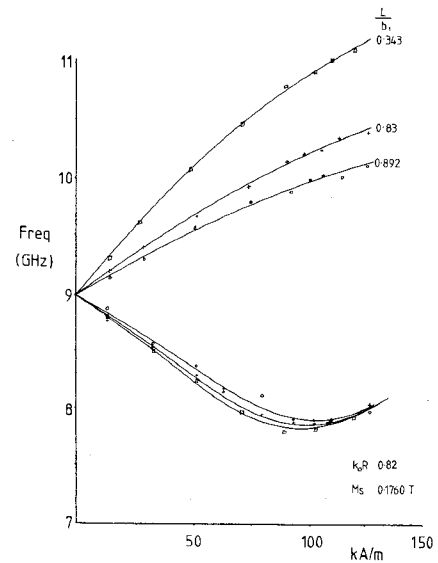


Fig. 3. Experimental split frequencies of loosely and tightly coupled, magnetized open ferrite disk resonators.

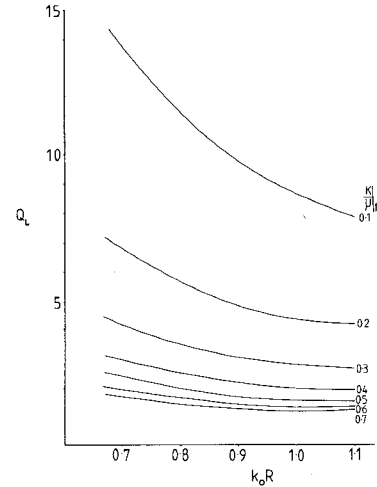


Fig. 4. Loaded  $Q$ -factor of open ferrite resonator versus  $k_0 R$  for parametric values of  $\kappa/\mu$ .

The most important quantity in the theory of quarter-wave coupled circulators is its loaded  $Q$ -factor. This parameter is usually expressed in terms of the split frequencies of the magnetized resonator using (1). The result is

$$Q_L = \left[ \sqrt{3} \left( \frac{\omega_+ - \omega_-}{\omega_0} \right) \right]^{-1}. \quad (15)$$

The loaded  $Q$ -factor of the junction is therefore readily evaluated from a knowledge of the split frequencies. Fig. 4 depicts the relationship between the loaded  $Q$ -factor and  $k_0 R$  for parametric values of the magnetic variable  $\kappa$ . Although the calculation of the split frequencies is exact, the derivation in (1) disregards the influence of higher order modes (in a strongly magnetized resonator) on the description of the gyrator circuit. The results in (1) and (15) therefore apply to a weakly magnetized resonator only. For the purpose of this paper, this condition is

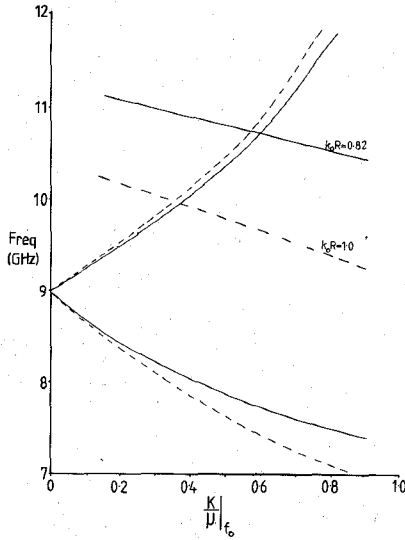


Fig. 5. Intersection of first two split resonance in magnetized resonator for  $k_0 R = 0.82$  and  $1.0$  and  $\epsilon_f = 15$ .

satisfied provided  $Q_L$  has a lower bound equal to approximately two. Such a value of loaded  $Q$ -factor is compatible with the performance of many commercial devices.

The basic approximation employed in this paper to describe the loaded  $Q$ -factor of the circulator is that in a weakly magnetized resonator only the first pair of counter-rotating modes need to be catered for in forming the complex gyrator circuit. A knowledge of the onset of the first higher order split pair of modes is therefore desirable. Fig. 5 depicts this result.

### III. SPLIT FREQUENCIES OF MAGNETIZED QUARTER-WAVE LONG OPEN RESONATOR LOADED BY IMAGE WALL

In a practical circulator arrangement, the open flat face of the resonator is loaded by an image or waveguide wall. The effect of this image wall on the frequencies of the demagnetized [8], [9], [16] and magnetized resonators is readily determined by satisfying the transverse resonance condition at the plane of the open flat face of the resonator. The model used here assumes that the open face of the resonator is enclosed by a contiguous magnetic wall waveguide below cutoff terminated by the image wall

$$\frac{\epsilon_f k_0}{\beta_0} \cot(\beta_0 L) - \frac{\epsilon_d k_0}{\alpha} \coth(\alpha \Delta s) = 0 \quad (16)$$

$$\frac{\epsilon_f k_0}{\beta_{\pm}} \cot(\beta_{\pm} L) - \frac{\epsilon_d k_0}{\alpha} \coth(\alpha \Delta s) = 0 \quad (17)$$

where

$$\alpha^2 = \left( \frac{1.84}{R} \right)^2 - (k_0 \sqrt{\epsilon_d})^2 \quad (18)$$

At a fixed frequency,  $L_0$  in (2) and (3) is now replaced by

$$L = L_0 - \Delta L \quad (19)$$

$\Delta L$  is a correction factor which accounts for the effect of

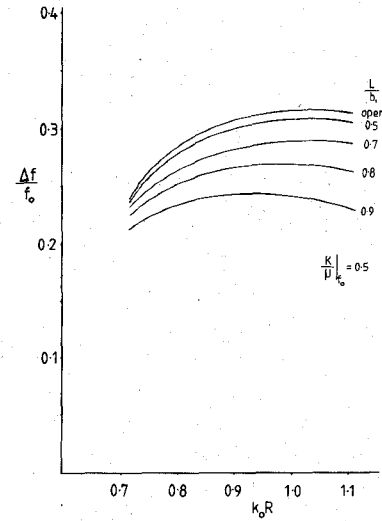


Fig. 6. Split frequencies of open ferrite resonator loaded by image wall versus  $k_0 R$  with  $\kappa/\mu = 0.5$  and parametric values of  $L/b_1$  for  $\epsilon_f = 15$ .

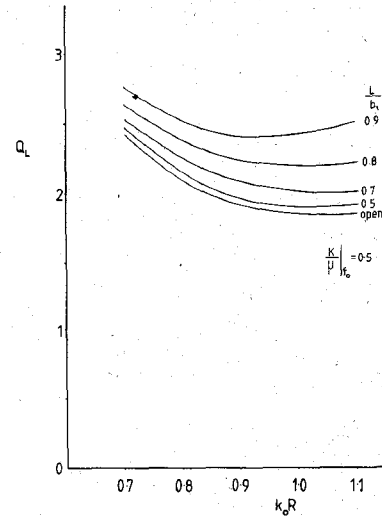


Fig. 7. Loaded  $Q$ -factor of open ferrite resonator loaded by image wall versus  $k_0 R$  with  $\kappa/\mu = 0.5$  and parametric values of  $L/b_1$  for  $\epsilon_f = 15$ .

the image or waveguide wall,  $\Delta s$  is the spacing between the resonator and the image wall,  $\epsilon_d$  is the relative dielectric constant of the region between the two. The first of these equations again fixes  $\beta_0 L$ , whereas the second may be evaluated for the two values of  $k_0$  corresponding to  $n = \pm 1$  in (8).

Figs. 6 and 7 depict the dependence of the split frequencies and the loaded  $Q$ -factor upon the location of the image wall described by

$$\frac{L}{b_1} = \frac{L}{L + \Delta S} \quad (20)$$

The results in these illustrations indicate that the solution of the open resonator represents an upper bound on the split frequencies and a lower bound on the loaded  $Q$ -factor of the junction. The frequency of the demagnetized resonator in (16) has been discussed in [8] and [9] and will not be considered here.

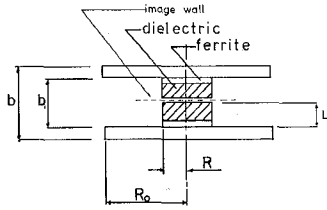


Fig. 8. Schematic diagram of waveguide circulator using open composite ferrite/dielectric resonator.

#### IV. SPLIT FREQUENCIES OF COMPOSITE RESONATORS

A resonator geometry that has some application in the design of large mean power devices is a composite resonator consisting of a ferrite/dielectric assembly [17]. The characteristic equation for the demagnetized and magnetized resonators (including the influence of the image wall) are readily formed as

$$k_0 \left[ \frac{1 - \frac{\beta_f \epsilon_r}{\beta_r \epsilon_f} \tan(\beta_f L_f) \tan(\beta_r L_r)}{\frac{\beta_r}{\epsilon_r} \tan(\beta_r L_r) + \frac{\beta_f}{\epsilon_f} \tan(\beta_f L_f)} \right] - \frac{\epsilon_d k_0}{\alpha} \coth(\alpha \Delta S) = 0 \quad (21)$$

$$\beta_r^2 = \left( k_0 \sqrt{\epsilon_r} \right)^2 - \left( \frac{1.84}{R} \right)^2 \quad (22)$$

where  $\alpha^2$  is given by (18) and  $\epsilon_r$  and  $\beta_r$  are the relative dielectric constant and phase constant of the dielectric region of the composite resonator.

For the demagnetized resonator  $\beta_f$  in (5) applies, whereas for the magnetized one  $\beta$  in (8) must be used.

The effect of the image wall may be discarded in the above equation by letting  $\Delta S$  approach infinity.

The composite resonator discussed here is depicted in Fig. 8. It is usually described in terms of a filling factor  $k_f$  given by

$$k_f = \frac{L_f}{L_f + L_r} = \frac{L_f}{L} \quad (23)$$

$L_f$  is the length of the ferrite section,  $L_r$  is that of the dielectric section, and  $L$  is the overall length.

The theoretical relationship between the filling factor  $k_f$  and the split frequencies of the composite resonator is depicted in Fig. 9 for various image plane locations  $L/b_1$ . Fig. 10 illustrates some experimental results for a resonator with  $M_0 = 0.1760$  T,  $\epsilon_f = 15.0$ ,  $\epsilon_r = 9.5$ . The slight shifts in the direct fields at which the resonators exhibit saturation is in keeping with the shape demagnetizing factors of the different resonators.

Fig. 11 indicates the loaded  $Q$ -factor for this junction.

#### V. SPLIT FREQUENCIES OF QUARTER-WAVE LONG OPEN TRIANGULAR RESONATOR

For completeness, the frequency splitting between the degenerate modes in an open magnetized ferrite resonator open-circuited at one end and short-circuited at the other

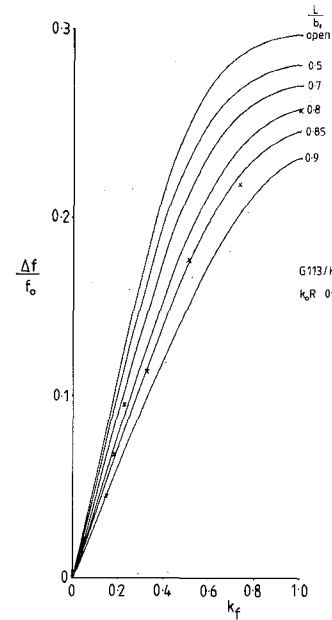


Fig. 9. Theoretical and experimental split frequencies of composite open resonator versus filling factor  $k_f$  for different values of  $L/b_1$  and  $\epsilon_f = 15$ ,  $\epsilon_d = 9.5$ .

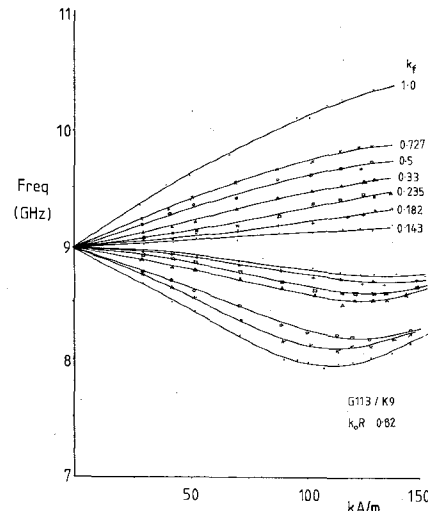


Fig. 10. Experimental split frequencies of composite resonator for different filling factors  $k_f$  and  $\epsilon_f = 15$ ,  $\epsilon_d = 9.5$ .

with a triangular cross section obtained using perturbation theory is reproduced below [19]

$$\frac{\omega_+ - \omega_-}{\omega_0} \approx \frac{\sqrt{3}}{\pi} \left( \frac{\kappa}{\mu} \right) \quad (24)$$

The operating frequency of the open resonator is

$$\left( \frac{\pi}{2L_0} \right)^2 = \left( k_0 \sqrt{\epsilon_f \mu_{\text{eff}}} \right)^2 - \left( \frac{4\pi}{3A} \right)^2 \quad (25)$$

$A$  is the side of the resonator,  $L_0$  is its length, and the other quantities have the meaning previously defined.

Fig. 12 depicts one experimental result at 9 GHz for a resonator with a magnetization of 0.1760 T, a dielectric constant of 15.0, an  $A$  dimension of 9.85 mm, and a height  $L_0$  of 3.25 mm. Although (24) has not been verified in the literature, it is compatible with the experimental work here.

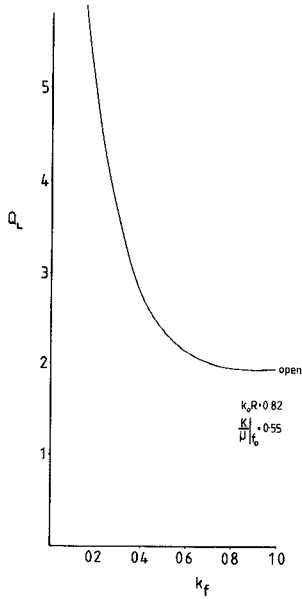


Fig. 11. Loaded  $Q$ -factor of open composite resonator versus filling factor  $k_f$  and  $\epsilon_f = 15$ ,  $\epsilon_d = 9.5$ .

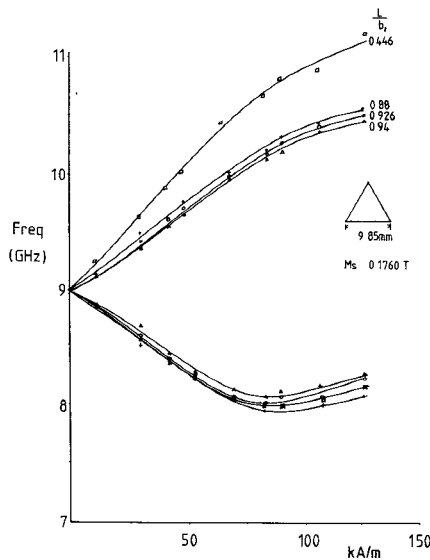


Fig. 12. Experimental split frequencies of open triangular ferrite resonator for different values of  $L/b_1$ .

The influence of the image wall on the frequency of the idealized open demagnetized resonator is obtained in the usual way by forming the transverse resonance relationship at the open flat plane of the resonator. The result is

$$\frac{\epsilon_f k_0}{\beta_0} \cot(\beta_0 L) - \frac{\epsilon_d k_0}{\alpha} \coth(\alpha \Delta S) = 0 \quad (26)$$

where

$$\alpha^2 = \left( \frac{4\pi}{3A} \right)^2 - (k_0 \sqrt{\epsilon_d})^2. \quad (27)$$

The frequencies of the magnetized resonator cannot be formed at this time because  $\beta_{\pm}$  are not known for this waveguide.

This result suggests that although weakly magnetized resonators with disk and triangular cross sections exhibit

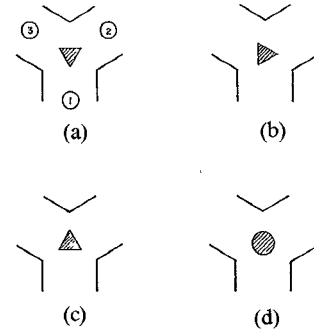


Fig. 13. Schematic diagrams of waveguide circulators using partial-height triangular resonators.

different susceptance slope parameters and gyrator levels, the loaded  $Q$ -factors of the two configurations are not very different. The choice between the two is therefore primarily determined by the network specification and the circuit configuration.

## VI. SUSCEPTANCE SLOPE PARAMETER OF DISK AND TRIANGULAR OPEN RESONATORS

In the ideal synthesis problem of quarter-wave coupled gyrator circuits, the rippled levels ( $S(\min)$ ,  $S(\max)$ ), and the normalized bandwidth ( $W$ ) are the independent variables, and the complex gyrator quantities ( $Q_L$ ,  $b'$ ,  $g$ ) are the dependent ones [14], [15]. However, in the class of devices considered here, the ripple levels and the loaded  $Q$ -factor (or susceptance slope parameter) are the independent ones, and the susceptance slope parameter (or loaded  $Q$ -factor) and the bandwidth are the dependent ones. A knowledge of the susceptance slope parameter is therefore necessary for a complete description of this class of device.

Although this work suggests that junctions using disk and triangular open resonators have similar loaded  $Q$ -factors, some preliminary experimental data in the cases of side and apex coupled triangular resonators indicates that their susceptance slope parameters differ [18]. The choice between the resonator geometries in Fig. 13 is therefore primarily determined by the ripple level. Unfortunately, a complete solution to this problem is not available. However, a semiempirical formulation of this problem in terms of the turns ratio of an ideal transformer has been described in the case of the disk resonator [9]. Since the susceptance slope parameter of the demagnetized triangular resonator has the same form as that of the disk one, it is therefore opportune and reasonable to describe their junctions in terms of semiempirical turns ratios also.

The result in the case of a single quarter-wave long disk resonator is [9]

$$B' = n^2 \zeta_0 \left\{ \frac{\pi \epsilon_r^2}{4} \left( \frac{k_0}{\beta_0} \right)^3 + \frac{\epsilon_r^2}{2} \left( \frac{k_0}{\beta_0} \right)^3 \left[ \tan \beta_0 \Delta L - \frac{\beta_0 \Delta L}{\cos^2 \beta_0 \Delta L} \right] + \frac{1}{2} \left( \frac{k_0}{\alpha} \right)^3 \left[ \coth \alpha S + \frac{\alpha S}{\sinh^2 \alpha S} \right] \right\} \quad (28)$$

where  $n^2$  is a semiexperimental turns ratio which is expressed in terms of the cross-sectional areas of the rectan-

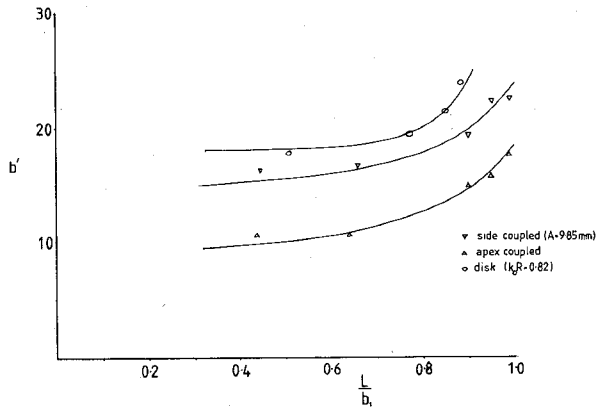


Fig. 14. Susceptance slope parameter of open disk and triangular resonators.

gular ( $ab$ ) and circular ( $\pi R^2$ ) waveguides

$$n^2 \approx \frac{1}{3} \left( \frac{ab}{\pi R^2} \right). \quad (29)$$

The susceptance slope parameters of the coupled disks and single cylinder versions are twice that of the single disk one [9]. The solution to the quarter-wave long triangular resonator is obtained by duality by replacing

$$kR = 1.84 \quad (30)$$

with

$$kA = \frac{4\pi}{3}. \quad (31)$$

and writing  $n^2$  on the basis of experiment as

$$n^2 \approx 0.12 \left( \frac{4ab}{\sqrt{3}A^2} \right), \quad \text{apex coupled triangle} \quad (32)$$

$$n^2 \approx 0.18 \left( \frac{4ab}{\sqrt{3}A^2} \right), \quad \text{side coupled triangle.} \quad (33)$$

$ab$  is the cross-sectional area of the rectangular waveguide and  $\sqrt{3}A^2/4$  is that of the triangular ferrite resonator.

Fig. 14 indicates some experimental results at 9 GHz in WR 90 waveguide. The susceptance slope parameters of typical disk resonators ( $k_0R = 0.82$ ) are included on this illustration for completeness. It is observed, in passing, that it is possible to have the susceptance slope parameter of the disk resonator overlap with either of the orientations of the triangular one by varying its radius.

The experimental points in Fig. 14 represent the average result obtained at the 3 ports of the junction.

## VII. FARADAY ROTATION

The operation of circulators using partial-height resonators is closely related to that of the classic turnstile circulator [1], [4], [5]. Since the latter is classically described in terms of Faraday rotation in a longitudinally magnetized circular waveguide, some qualitative attempt to describe the former in the same terms has been made [7]. However, no satisfactory quantitative correspondence between the two has been established to date. The correspondence

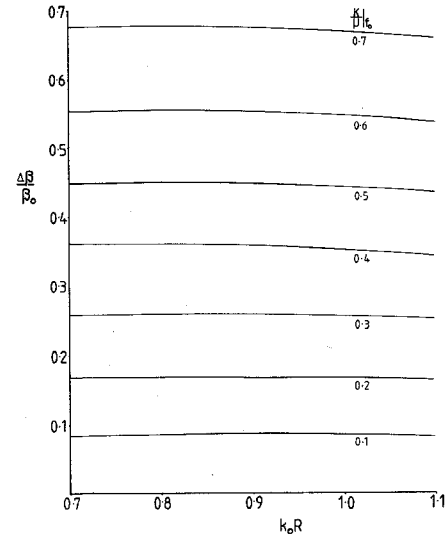


Fig. 15. Split phase constants of magnetized open-ferrite disk resonator for  $\epsilon_f = 15$ .

between them may be derived by forming the split phase constants in (8). This result is shown in Fig. 15. One approximate relationship between the split phase constants of the magnetized ferrite waveguide and the magnetic variables for  $k_0R = 0.8$  is

$$\frac{\beta_+ - \beta_-}{\beta_0} \approx 0.356 \left( \frac{\kappa}{\mu} \right)^3 - 0.101 \left( \frac{\kappa}{\mu} \right)^2 + 0.855 \left( \frac{\kappa}{\mu} \right), \quad 0 \leq \left( \frac{\kappa}{\mu} \right) \leq 0.5. \quad (34)$$

Combining (14) and the preceding equation leads to

$$\frac{\omega_+ - \omega_-}{\omega_0} \approx \frac{2}{3} \frac{\beta_+ - \beta_-}{\beta_0}, \quad 0 \leq \left( \frac{\kappa}{\mu} \right) \leq 0.5. \quad (35)$$

An equivalent representation of the gyrator conductance of waveguide circulators using partial-height open resonators is therefore

$$g \approx \frac{2}{\sqrt{3}} b' \left( \frac{\beta_+ - \beta_-}{\beta_0} \right), \quad 0 \leq \left( \frac{\kappa}{\mu} \right) \leq 0.5. \quad (36)$$

The loaded  $Q$ -factor of such a junction may also be described from the preceding equation by

$$\frac{1}{Q_L} \approx \frac{2}{\sqrt{3}} \left( \frac{\beta_+ - \beta_-}{\beta_0} \right), \quad 0 \leq \left( \frac{\kappa}{\mu} \right) \leq 0.5. \quad (37)$$

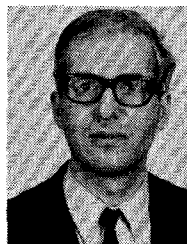
## VIII. CONCLUSIONS

A knowledge of the loaded  $Q$ -factor and susceptance slope parameter of a junction circulator is sufficient for the synthesis of quarter-wave coupled devices. For weakly magnetized resonators the former quantity may be expressed in terms of the split frequencies of the magnetized resonator. This is done in this paper for a number of cylindrical resonators assuming that the side walls may be idealized by a magnetic wall boundary condition. The agreement between theory and experiment is excellent. Experimental data on the split frequencies of magnetized

triangular resonators are also given. Some remarks about the susceptance slope parameters of disk and triangular resonators are included for completeness.

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